NONLINEAR ELECTRIC FIELD OSCILLATIONS IN CONTINUOUS MEDIA

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Strong nonlinear potential and drift-velocity waves in a nonequilibrium medium, in which the current-carrier collision frequency depends on the electric field, are considered in the hydrodynamic approximation.

Under certain conditions, the differential operator describing the electric field in a nonlinear medium can change from one type to another [1]. In this case field oscillations can appear which cannot be treated in the context of a linear theory, because the variations in the steady state of the initially undisturbed quantities are not negligible. For a logical examination of the structure of high-amplitude waves in an electric field, the kinetic equation for the current carriers, as well as the field equations, must be used. Such a study of the oscillations is very detailed, and it is extremely complicated from the mathematical point of view. An exact solution of the problem is easily obtained on the basis of the equations of plasma dynamics in the hydrodynamic approximation.

The equations of motion and continuity for the electrons and Poisson's equation for the electrostatic potential provide our starting-point in the problem. Conditions are considered for which the natural magnetic field of the current and the charge pressure gradient can be neglected.

1. In the one-dimensional nonstationary case, for a fixed ion background, we have the system of equations

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{e}{m}\frac{\partial \varphi}{\partial x}+\frac{u}{\tau}=0,\quad \frac{\partial n}{\partial t}+n\frac{\partial u}{\partial x}+u\frac{\partial n}{\partial x}=0,\quad \frac{\partial^2\varphi}{\partial x^2}=\frac{4\pi e}{\varepsilon}(n_0-n) \qquad (1.1)$$

where u, e, m, and τ are the directed velocity, charge, mass, and collision time of an electron, respectively; φ is the electrostatic potential; n and n_0 are the electron and ion densities; and ε is the dielectric constant.

The nonlinearity of system (1.1) is due to the presence of the delay term udu/dx in the equations of motion and the dependence of the relaxation time τ on the velocity u. A general investigation of (1.1) is possible without specifying a particular function $\tau = \tau(u)$; it is only necessary for $\tau(u)$ to be smooth, continuous, and increasing.

We consider the solution of (1.1) in a coordinate system moving with constant velocity U relative to the initial system ($\xi = x - Ut$). In the moving system

$$(u-U)u'+\frac{e}{m}\varphi'+\frac{u}{\tau}=0,\qquad (u-U)n=\text{const}=(u_0-U)n_0$$
$$\varphi''=\frac{4\pi e}{\varepsilon}(n_0-n) \qquad (1.2)$$

where the primes denote differentiation with respect to ξ.

Eliminating the potential from the first equation, we obtain a nonlinear second-order equation for the velocity:

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$$(u - U)\, u'' + (u')^2 + \frac{1 - \Lambda_\tau^2}{\tau} u' + \omega_0^2 \frac{u - u_0}{u - U} = 0$$
$$\Lambda_\tau = \frac{u}{U_\tau}, \quad U_\tau^2 = u\tau \frac{\partial u}{d\tau}, \quad \omega_0^2 = \frac{4\pi e^2 n_0}{m\varepsilon} \tag{1.3}$$

Here, ω_0 is the plasma frequency; Λ_τ is a dimensionless number; U_τ is the local velocity of propagation of disturbances in the nonlinear medium; and U_τ and Λ_τ characterize, respectively, the absolute and relative degrees of medium nonlinearity, due to the dependence of τ on u.

If U_τ takes values $U_\tau < u$ ($\Lambda_\tau > 1$), then it follows from

$$1 - \Lambda_\tau^2 = 1 - \frac{u}{\tau}\frac{d\tau}{du} = \tau \frac{d}{du}\frac{u}{\tau} \tag{1.4}$$

that, in the assumed conditions, the coefficient of dynamic friction u/τ will increase along with u, until the latter reaches a critical value u_*, corresponding to $\Lambda_\tau = 1$; as u increases beyond this critical velocity, the coefficient of friction decreases (Fig. 1).

On the whole, the coefficient

$$\omega_d = \frac{1 - \Lambda_\tau^2}{\tau} = \frac{d}{du}\frac{u}{\tau}$$

of u' in (1.3) is a differential frequency, which is positive when $\Lambda_\tau < 1$ and takes negative values when $\Lambda_\tau > 1$ (Fig. 2). The ratios of the absolute values of the plasma frequency ω_0, the differential frequency ω_d, and the collision frequency $\omega = 1/\tau$ determine the nature of the nonlinear oscillations of the drift velocity, the electron density, and the electrostatic potential.

2. Equation (1.3) has one simple equilibrium state $(u_0, 0)$ on the phase plane u, u', while the roots of the characteristic equation are given by

$$\lambda_{1,2} = \frac{1}{u_0 - U}\left[-\frac{1 - \Lambda_\tau^2}{2\tau} \pm \left(\frac{1 - 2\Lambda_\tau^2 + \Lambda_\tau^4}{4\tau^2} - \omega_0^2\right)^{1/2}\right] = \frac{1}{u - U}\left[-\frac{\omega_d}{2} \pm \left(\frac{\omega_d^2}{4} - \omega_0^2\right)^{1/2}\right] \tag{2.1}$$

Seven types of singularity are possible:

a) a simple stable node $0.5\omega_d > \omega_0$,
b) a degenerate stable node $0.5\omega_d = \omega_0$,
c) a stable focus $0.5\omega_d < \omega_0$,
d) a center $\omega_d = 0$,
e) an unstable focus $-0.5\omega_d < \omega_0$,
f) an unstable degenerate node $-0.5\omega_d = \omega_0$,
g) a simple unstable node $-0.5\omega_d > \omega_0$.

A diagram showing the regions of the $\omega_* = 1/\tau\omega_0$, Λ_τ plane in which the different types of singularity exist is given in Fig. 3; in 1 and 2 we have a stable node and focus, and in 3 and 4, an unstable focus and node, respectively. On the boundaries of node and focus regions we have degenerate nodes, and on the boundary of the stable and unstable focus regions we have a center.

Thus, if the directed electron velocity u is greater than the velocity U_τ ($\Lambda_\tau > 1$), unstable oscillations appear in the medium. This is connected with the fact that the friction force mu/τ decreases as u increases. When $u < U_\tau$ ($\Lambda_\tau < 1$), the friction increases with u (Fig. 1), so that the velocity oscillations are always damped. When $u = U_\tau$ ($\Lambda_\tau = 1$), the variation of the friction force is zero, or more precisely, it is independent of the velocity ($\tau = \alpha u$, α = const), and a periodic oscillation of u occurs in the medium as a result of the inertia in the electron movement.

In this case, the first integral of (1.3) is

$$(u - U)u' = \omega_0 [C^2 - (u - u_0)^2]^{1/2} \tag{2.2}$$

where the constant C defines the amplitude of oscillations. On integrating (2.2) we get

$$(u_0 - U) \arcsin \frac{u - u_0}{C} - [C^2 - (u - u_0)^2]^{1/2} = \omega_0 (x - Ut) \tag{2.3}$$

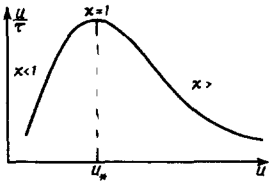

Fig. 1

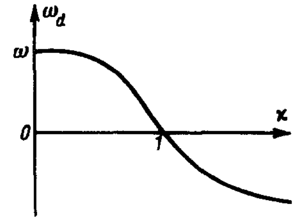

Fig. 2

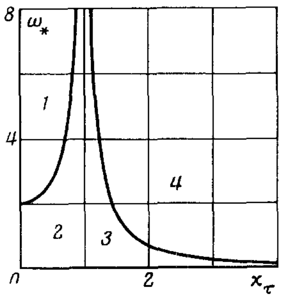

Fig. 3

We see that the periodic oscillations of the velocity in the nonlinear medium have a frequency equal to the plasma frequency ω_0.

Successively integrating Poisson's equation, we get

$$\frac{e}{m}\varphi' = -\alpha - \omega_0\left[C^2 - (u-u_0)^2\right]^{1/2} \tag{2.4}$$

$$(u_0 - U)\arcsin\left[\frac{1}{C}\left(\sqrt{2\alpha\xi + 2\frac{e}{m}\varphi} - u_0 + U\right)\right]$$

$$-\left[C^2 - \left(\sqrt{2\alpha\xi + 2\frac{e}{m}\varphi} - u_0 + U\right)^2\right]^{1/2} = \omega_0\xi$$

This shows that the linear variation of the potential in ξ is modulated by oscillations of the plasma frequency. When $\alpha = 0$ (no collisions), Eq. (2.4) yields the expressions obtained by Akhiezer and Lyubarskii concerning the nonlinear oscillations of a collision-free cold plasma [2].

LITERATURE CITED

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2. A. I. Akhiezer and G. Ya. Lyubarskii, "Nonlinear theory of electron plasma oscillations," Dokl. AN SSSR, vol. 80, pp. 193-195, 1951.